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Cole: Cops and Robbers

Cops and Robbers Looking at the Interdependent Relationship between Police and Crime Using Differential Equations

Matthew T. Cole

ABSTRACT. This paper devises a system of differential equations to model the dual dependency of police and crime, based on a predator/prey relationship. It emphasizes the complexity of the relationship and shows how empirical research can be skewed if it only focuses on a limited time interval. The system of differential equations explains why some researchers have come to the mistaken conclusion that hiring more police induces more crime, and gives policy makers better information with which to make decisions.

I. Introduction

Which came first, the chicken or the egg? This has been a long debated philosophical question. It was designed to make people think, and then realize that not every question has a clear-cut answer. At first glance, one might reason that chickens come from eggs; therefore the egg must have come first. But, after further thought, one realizes that eggs come from chickens. Therefore, the chicken must have come first. The question can be a bit frustrating.

The same reasoning can be used when investigating the relationship between crime and police. On one hand, the amount of crime influences the number of police officers employed. But, on the other hand, the number of police officers influences the amount of crime. The question is not one of which came first, but what influences what. The answer is, they both influence each other.

As the number of crimes rise, citizens are more apt to increase funding of the police force so as to hire more officers. Similarly, if the amount of crime decreases, the citizens would rather spend their tax money elsewhere. The size of the police force, therefore, is dependent on the amount of crime. At the same time, the size of the police force influences the amount of crime. "Police reduce crime either via deterrence (preventing the commission of the initial crime due to an increased likelihood of being caught), or through incapacitation (catching repeat offenders so they cannot commit future crimes)" [Levitt, 1997,

278]. Therefore with more police, crime is reduced. Consequently, the occurrence of crime is dependent on the size of the police force. There is a dual dependency between the two.

Because of the dual dependency, this relationship can be difficult to model with linear regression. Although econometrics can be used to model dual relationships, there may be another way. This other way is by using a system of differential equations. The system of differential equations derived will be very basic, but will provide useful information for understanding the relationship between police and criminals. With this understanding, public officials will be better equipped to make policy decisions.

II. Background

To better understand the model about to be presented, some mathematics needs to be explained. A derivative is the rate of change of a given variable with respect to some other variable. This is sometimes referred to as the marginal of that variable (e.g. marginal cost, marginal revenue, etc.). “A differential equation is a relationship between a function of time and its derivatives [Braun, 1993, 1].” This equation can be denoted as

$$\frac{dC(t)}{dt}$$

read as the derivative of $C(t)$ with respect to time (t) . For the purposes of this paper, it is the rate of change in crime with respect to time.

A system of differential equations is two or more equations where the rate of change of one variable depends not only on time and itself, but on the value of all the other variables as well [Braun, 1993, 265]. The system of equations that will be devised is based on a similar scenario dealing with a predator and prey relationship found in Blanchard, Devaney and Hall’s textbook, *Differential Equations* [1998, 144]. In this paper, criminals can be thought of as the prey and the police as predators.

Throughout this paper, it should be noted that I am taking an economic approach to crime and crime prevention.

The economic approach assumes that both criminals and victims are rational in the sense that they base their choices on the expected benefits and costs of alternatives. A rational person commits a property crime (*defined by the author, as crimes of*

stealth, not force. Examples are burglary, larceny, and auto theft.) if the expected benefit of the crime exceeds the expected cost. Similarly, potential victims use their resources to prevent crime if the expected benefit of prevention exceeds the expected cost. The economic approach leads logically to a discussion of the optimum amount of crime: because crime is costly to prevent, it is rational to allow some crime to occur [O'Sullivan, 1996, 651-2].

O'Sullivan goes on to state, "The model of the rational criminal is relevant for economically motivated crimes (property crimes), but not for crimes of passion and violence [O'Sullivan, 1996, 657]." This paper, therefore, addresses only economically motivated crimes.

III. The Model

To begin, assume that there are no police and that we are living in a society with a relatively low occurrence of crime. We need to start with this assumption in order to better understand the characteristics of the function. Intuitively, it is safe to say that crime will increase in the absence of police officers. The question is, will crime rise at a steady rate or will the rate of change differ at different points in time? To answer this question we need to look at the psychology of crime.

A study by James Wilson and George Kelling looks at the effects of disorder on a community. They note that

Social Psychologists and police officers tend to agree that if a window in a building is broken and is left unrepaired, all the rest of the windows will soon be broken...One unrepaired broken window is a signal that no one cares, and so breaking more windows costs nothing...Untended property becomes fair game for people out for fun or plunder, and even for people who ordinarily would not dream of doing such things and who probably consider themselves law-abiding [Wilson and Kelling, 1982, 31].

In other words, an area that is in disorder is more likely to see more of an increase in crime than a similar area that has order. In support of this, Wesley Skogan states "robbery rates and other indicators of the extent of neighborhood crime are strongly related to the level of perceived

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disorder” [1990, 48]. Therefore, the connection can be made that crime implies disorder, which induces robbery and other neighborhood crimes. Thus, crime induces (at least indirectly) more crime.

This relationship is expressed by the equation

$$\frac{dC(t)}{dt} = \alpha C(t)$$

where α is a positive constant coefficient and $C(t)$ is the number of crimes at a point in time. This equation states that the amount of change in crime is dependent on the amount of crime that already exists. This means that *ceteris paribus* the occurrences of crime would increase at an increasing rate.

The graphs corresponding to this differential equation for crime are as follows:

But, is this an accurate picture? For instance, we know that crime increases, but does it increase exponentially or logarithmically? In other words, will crime continue to rise forever or is there a maximum number of offenses possible at a given point in time?

To answer this, the mathematics of the situation needs to be explained more. Remember that a differential equation is a relationship between a function of time and its derivatives. The key word here is time.

Consequently, one criminal can only commit one crime at a given point in time.

Now, imagine the worst-case scenario. Suppose there is a town with a population of 100 people and everyone in this town is a criminal. Logically, only 100 crimes could be committed at a given time. To better understand, place everyone in a circle and have each person steal from the person directly to his or her left. One person could not steal from two people simultaneously.

Human behavior also needs to be explored. “Most people have an underlying aversion to committing crime. For the typical person, crime is unattractive at any price [O’Sullivan, 1996, 659].” Therefore, even without the presence of police, there will be citizens who will not commit crimes.

The crime equation must have a logarithmic pattern of growth with a horizontal asymptote. This asymptote should be at the level of the population minus the number of people with an aversion to committing crime. However, for simplicity, only the population will be used. The logarithmic pattern is represented by

$$1 - \frac{C(t)}{\varpi}$$

where ϖ is the population. Without this term, the model would imply that crime could increase to infinity. Moreover, crime occurrences could exceed the size of the population at a given point in time.

To understand how this term puts an upper limit on crime, just look at what happens as crime increases. As the number of crimes approaches the population, the term $\{C(t)/\varpi\}$ approaches 1; the change in crime then becomes zero. Note that the amount of crime is not zero, just the change in crime. With this added term, the differential equation for crime is a bit more complicated but more realistic.

$$\frac{dC(t)}{dt} = \alpha C(t) \left(1 - \frac{C(t)}{\varpi} \right)$$

Now, let us address the effects police have on crime. Before this can be done, it is prudent to make sure the number of police officers does in reality have an effect on crime. Erling Eide approaches this problem by looking at the individual's expected utility from committing an offense. Eide presents the following equation:

$$E[U] = PU(W-f) + (1-P)U(W+g)$$

Where $U(.)$ is the individual's utility function... P is the subjective probability of being caught and convicted... f is the monetary equivalent of the punishment... W is present income and g is gains from crime [Eide, 1997, 3]

It would seem reasonable that adding more police officers would increase the probability (P) of being caught and convicted. This increase in ' P ' would cause the expected utility for some individuals to become negative, implying that a rational individual would not commit the crime. The person will expect to be worse off by committing the crime.

For empirical evidence we can look to Steven Levitt's, *Using Electoral Cycles in Police Hiring*. He states, "The results do provide evidence suggesting that additional police reduce crime." However, he also states, "the imprecision of the estimates makes it impossible to reject the null hypothesis that the current level of police is set optimally in large

cities” [Levitt, 1997, 286]. Further insight can be found in a paper released by The Fraser Institute, which says that

Evidence from the United States suggests that once the effect of more crime in enlarging the police complement is taken into account, an increase of one police officer reduces crime by 8 to 10 events per year. These events are spread across the categories of murder, rape, assault, robbery, burglary, larceny, and auto theft...one additional officer reduces auto theft by 5 to 7 vehicles per year [The Fraser Institute, 1999, para. 3]

There is still much debate on whether police, in fact, have an effect on crime. Some have even said that adding police actually increases crime rates. “Of the 22 studies surveyed by Samuel Cameron (1988) that attempt to estimate a direct relationship between police and crime using variation across cities, 18 find either no relationship or a positive (i.e. incorrectly signed) relationship between the two [Levitt, 1997, 270].” This theory will be examined in more detail later in the paper. However, for now it seems safe to assume that police do have a negative effect on crime. This adds the term $-\beta P(t)$ to our differential equation for crime, as follows.

$$\frac{dC(t)}{dt} = \alpha C(t) \left(1 - \frac{C(t)}{\varpi} \right) - \beta P(t)$$

Where β is a positive constant coefficient and $P(t)$ is the number of police officers at a point in time.

The number of police officers, though, is not the only factor that has a negative effect on the occurrences of crime. As mentioned earlier, the amount of crime has a positive effect on the rate of change in crime. However, it also has a negative effect as well. As crime becomes more rampant, the profitability of crime goes down [McCormick, 2001]. With more criminals comes more competition for that nice Lexus you want to steal. Therefore, maybe you will not be able to steal that Lexus, but be forced to steal only a Dodge instead. The added competition causes the expected profit from your crime to decrease.

It's important to note that although $C(t)$ has both a positive and negative effect, the negative effect is more apparent when $C(t)$ is large.

In other words, when $C(t)$ is small the positive effects are larger than the negative effects, and the opposite is true when $C(t)$ is large.

In the end, the total negative effect on the number of crimes is represented by $\beta C(t)P(t)$, and the final equation for the rate of change in crime is:

$$\frac{dC(t)}{dt} = \alpha C(t) \left(1 - \frac{C(t)}{\varpi} \right) - \beta C(t)P(t)$$

The reason $\beta C(t)P(t)$ is multiplied together as opposed to having two separate entities is for simplicity. For example, if $\beta C(t) - \delta P(t)$ was used instead, a theoretical problem would exist. If crime were zero, the change in crime would be equal to $-\delta P(t)$, a negative amount. This would imply that there is a decrease in crime after there is already no crime, which is impossible.

We now need the second component of the system, an equation that measures the change in the number of police officers over time. Assume for the sake of argument that all of a sudden there is no crime. It is safe to believe that in the absence of crime the number of police officers will go down, because without crime people will not want to spend money on an unneeded service. But would the size of the police force decrease at a constant rate? The answer would seem to be no. It is reasonable to assume that the greater the number of police officers in an area without crime, the greater would be the decrease in the number of police officers in that area.

For example, suppose a town has 50 cops, and all at once (however unlikely) crime stops. A rational citizen would get tired of paying taxes that go for unneeded police officers and might decrease the size of the police force by, say, 25. Now, the town has 25 officers and still no crime. The citizens still believe they are paying for too many officers. However, it is reasonable to assume that the town would not decrease the number of officers by another 25. This time they might get rid of, say, 15. Logically, if the town kept reducing the size of the police force by 25, soon the size would be zero. Furthermore, the size would keep decreasing until it was negative, which is impossible. Therefore, the change in the size of the police force over time is influenced by the size of the police force, and is equal to $-\psi P(t)$, where ψ is a positive constant coefficient. In other words,

$$\frac{dR(t)}{dt} = -\psi P(t)$$

This picture, though, is not quite accurate. It would seem reasonable that even without crime, a society would want a minimum number of police officers. There are two major reasons why people may want this minimum. The first is the uncertainty about future crime and the desire for basic prevention of that crime. A second reason is the diverse job description of a police officer. The job of a police officer is not only to fight crime. According to the American Bar Association, the major current responsibilities of police are as follows:

- i. To identify criminal offenders and criminal activity and, where appropriate, to apprehend offenders and participate in subsequent court proceedings;
- ii. To reduce the opportunities for the commission of some crimes through preventive patrol and other measures;
- iii. To aid individuals who are in danger of physical harm;
- iv. To protect constitutional guarantees;
- v. To facilitate the movement of people and vehicles;

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- vi. To assist those who cannot care for themselves;
- vii. To resolve conflict;
- viii. To identify problems that are potential serious law enforcement or governmental problems;
- ix. To create and maintain a feeling of security in the community;
- x. To promote and preserve civil order; and
- xi. To provide services on an emergency basis [1980, 3-4].

So even in the absence of crime, society has a need for a minimum number of police officers. Of the previous list, only (i) directly deals with crime prevention, and (ii), (vii), (ix), and (x) prevent crime indirectly. The remaining six responsibilities on the list show that police are needed even without crime. Thus, there is a lower limit put on the possible number of police officers.

Let λ be the lower limit on the number of police officers, adding the term $(1 - \{P(t)/\lambda\})$ to the equation. The number of police will always be greater than this lower limit, and as it approaches this limit, the term will go to zero. Moreover, this term is always negative when it is not zero. Therefore, the negative sign on ψ is dropped.

When this term reaches zero, the change in the number of police officers (in the absence of crime) will become zero. This results in a constant number of police officers equal to λ and we are left with the following equation:

$$\frac{dP(t)}{dt} = \psi P(t) \left(1 - \frac{P(t)}{\lambda} \right)$$

Before we can bring the effects of crime into the equation, there is a problem that must be addressed. Since $P(t)$ is always greater than λ , $\{P(t)/\lambda\}$ is always greater than 1. Thus, the change in the number of police officers would be negative and greater than the number of police officers currently employed. This would give us a negative police force, which is impossible.

Note that a constant coefficient (ψ) will not fix the problem. This is because the further away $P(t)$ is from λ , the smaller the coefficient would need to be to counteract the effects. To eliminate the problem, the coefficient can no longer be a constant, but must depend on $P(t)$ and λ . In other words,

$$\psi = \frac{1}{v(P(t) - \lambda)}$$

where v is a positive constant coefficient between zero and one. The further $P(t)$ is away from λ , the larger the denominator gets. Hence, the greater the difference between $P(t)$ and λ , the smaller ψ is.

The reason λ is used is as follows. As $P(t)$ approaches λ , the denominator approaches zero and the equation becomes undefined. The differential equation, however, will already be approaching zero at the same time because of the lower limit term. Therefore, anything greater than λ should not be used because the equation would become undefined before the equation becomes zero. In reality, perhaps a number less than λ should be used. However, for simplicity, λ will be used.

Now, we can finally incorporate crime into the equation. *Ceteris paribus*, there is a positive relationship between the number of crimes

committed and the change in the size of the police force. In other words, the more crime there is, the more police officers a community would be willing to hire. Hence, this will add the term $\phi C(t)$ to the equation, where ϕ is a positive constant coefficient.

We are thus left with the following system of equations:

$$\begin{aligned}\frac{dC(t)}{dt} &= \alpha C(t) \left(1 - \frac{C(t)}{\varpi} \right) - \beta C(t) P(t) \\ \frac{dP(t)}{dt} &= \left(\frac{1}{\nu(P(t) - \lambda)} \right) P(t) \left(1 - \frac{P(t)}{\lambda} \right) + \phi C(t)\end{aligned}$$

One possible graph for this system is as follows:

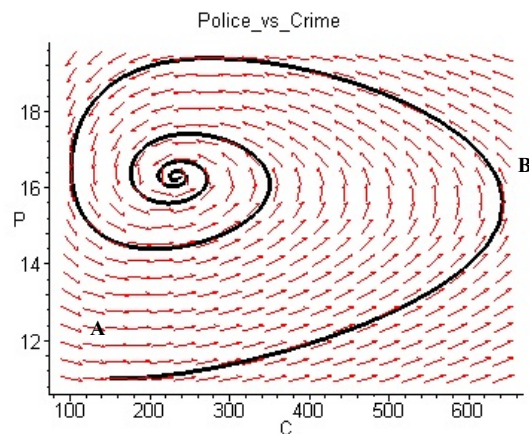


FIGURE 1.

This is a direction field. Given an initial condition, it plots both police and crime over time. Keep in mind that this is not a real life example. Gathering the actual data needed to estimate each of the coefficients in the model accurately would be quite extensive, and is beyond the scope of this paper.

The coefficients used for this particular direction field were manipulated to get what I believe to be a realistic looking direction field. They were derived under the assumption that the number of police officers (4,805) and acts of larceny (71,301) for the state of Iowa in 1997

were already at an equilibrium [FBI, 1997, 303 & 71]. The number of police officers and acts of larceny were then divided by the population to obtain a ratio. The data were then manipulated to fit an area of 10,000 people with a minimum number of officers equal to 10.

IV. Interpretation

Now that we have a direction field, we can address a theory that was mentioned earlier in the paper, namely the counter-intuitive idea that adding police causes crime to increase. When following the curve from point A to point B, it is easy to see where one could come to this conclusion. With an initial condition at $P(0)=11$ and $C(0)=150$, the number of police and crimes are both increasing. So, if the researcher were looking at a society during this stage, it would seem reasonable to conclude that the police have a positive effect on crime. After a certain point, though, the police size still increases but the amount of crime begins to decrease. This would imply an inverse effect. This back and forth continues until there is equilibrium between crime and police.

The two main forces driving the system towards its equilibrium are the criminals' fear of being caught and the society's willingness to pay to have order. The criminals' fear of being caught depends on their expected costs, which in turn depends on the probability of arrest and imprisonment, as well as the severity of punishment [O'Sullivan, 1996, 661] and the attitude of the criminal towards risk (which is very difficult to alter with public policy).

By adding more officers, the probability of getting caught increases. However, without a sufficient chance of punishment, the arrest brings no real consequences and even a 100% probability of arrest will have little effect. Therefore, increasing the probability of punishment raises the expected cost to the criminal and will move the point of equilibrium either to the left, down, or a combination of the two. Each scenario brings equilibrium closer to the origin.

Similarly, increasing the severity of punishment induces the same results, but not to the same extent. "There is evidence that an increase in the certainty of punishment provides a greater deterrent than an increase in the severity of punishment [O'Sullivan, 1996, 675]." In addition, when dealing with the question of how to increase the penalty for a crime, the policy makers need to examine any substitution effect. As O'Sullivan

states, “because stiffer penalties provide a marginal deterrent, an increase in the penalty for one crime causes some criminals to upgrade to a higher level crime [1996, 674].”

There is another point that needs to be addressed. The probability of arrest is not controlled solely by public policy; the criminal has some control over this as well. Assume, for example, that the Mafia or some other criminal organization moves into a community. The number of police officers stays the same, but because of the organization of the new criminals, the probability of their arrest decreases. With this new organization, the same number of officers does not have the same effect on crime as before. In terms of our equations, β has decreased. The more organized criminal has moved the point of equilibrium further away from the origin. Keep in mind that when the society initially moves away from equilibrium, this is not the new equilibrium point. The new point ($C(0) = 300$ & $P(0) = 16$) is an initial condition for the new system of equation and will follow a new direction curve to a new equilibrium (See figure 2).

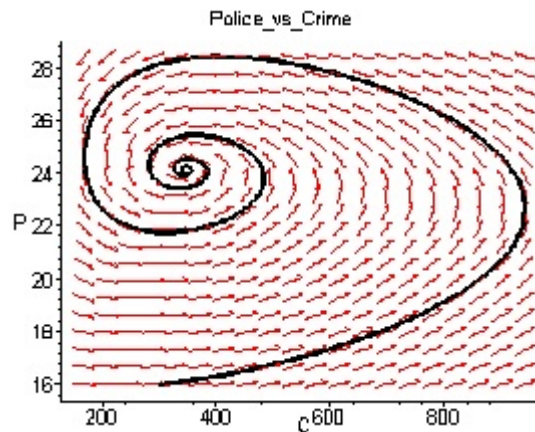


FIGURE 2.

Again, it is important to note that a direction field is not a graph similar to your typical supply and demand graphs. When labor becomes more efficient, the supply of the output curve shifts to the right. With a direction field, on the other hand, when criminals become more organized, the whole system changes. In other words, given the same initial conditions, the equilibrium is different as well as the path to

equilibrium. The curve is not merely shifted in a particular direction, but changed.

If, on the other hand, the increased crime had nothing to do with an improved criminal, the society would still move away from the equilibrium point (from point E to D in figure 3). However, over time, the society would return to the original equilibrium.

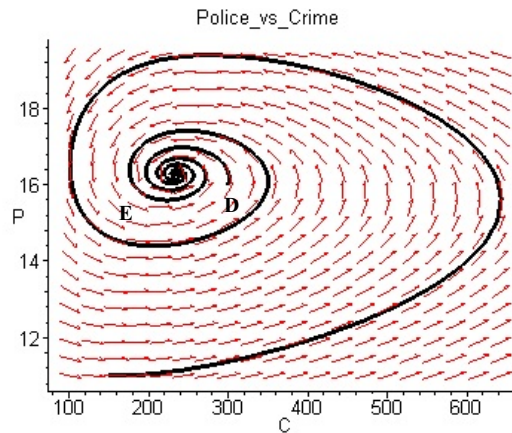


FIGURE 3.

Similarly, if the police were to have a “lucky streak” and reduce crime, the society would move off equilibrium for a short period of time, but return to the same point. Only if the police became more efficient would the actual equilibrium change.

V. Conclusion

Like all relationships, the interaction between police and criminals is dynamic. This paper has shown that although police have a negative effect on crime, during certain time intervals, it can appear to have a positive effect. In order to fully understand how police affects crime and vice versa, the whole picture needs to be examined. As shown, differential equations can be a good tool to accomplish this.

Though the model derived in this paper is very basic, it provides useful information for understanding the relationship between “cops and robbers.” With this understanding, public officials will be better equipped to make policy decisions. I hope that the model will be a

stepping-stone for future research. One suggestion to future researchers would be to incorporate lags into the system. For instance, an increase in police in $t = 1$ might have its significant effect in $t = 3$. Similarly, citizens might view an increase in crime as a one-time occurrence and could take more time to implement an appropriate response.

As the study of economics grows and we discover the increasing complexities of the relationships studied, the tools we use need to grow as well. The purpose of this paper was to provide an alternate tool to study the interaction between the size of the police force and the crime rate. As mentioned earlier, the model has much room for improvement. It has, however, accomplished its purpose, which is to provide a new way of looking at a long debated problem.

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